Are Professional Forecasters Bayesian?

Online Appendix

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1 Construction of pseudo-histograms

This Section provides a detail discussion of the construction of the pseudo histograms when forecasters assign a large probability to the open intervals of the Surveys.

Probability on two or more intervals When the truncation occurs in the US-SPF and ECB-SPF, the most common situation is that forecasters assign probability to the lowest open interval, but also to the adjacent closed intervals. This occurred for 54 of the 57 forecasters that contributed to the ECB-SPF in the first quarter of 2009. In this situation, I define the parameter b of the pseudo triangular distribution to be equal to the upper bound of the rightmost interval. The second assumption is that the mode c is equal to the point forecast provided by the forecaster. Given the values of b and c it is then possible to obtain the parameter a as the value such that the probability of the interval $[a, -1)$ based on the triangular distribution equals the probability assigned by the forecaster to the open interval, which we denote by $\tilde{p}$. This requires to distinguish the cases when c is larger or larger than the truncation point -1. If $c > -1$, a is obtained by solving the quadratic equation $\tilde{p} = \frac{(1+a)^2}{(b-a)(c-a)}$ and selecting the solution such that $a < -1$. As it was mentioned earlier, for the ECB-SPF Survey in the third quarter of 2009 there were only 5 forecasters that reported a point forecast that was more optimistic than -1%. On the other hand, for the remaining forecasters that reported a point forecast smaller relative to the truncation point, a is obtained as the solution to the equation $\tilde{p} = 1 - \frac{(1+b)^2}{(b-a)(b-c)}$ which is given by

$$a = b - \frac{(1+b)^2}{(1-\tilde{p})(b-c)}$$

The parameters a, b, and c completely characterize the triangular distribution which I then use to construct the pseudo histogram based on the grid of values used in the following quarter. In some cases, the value of a obtained from the Equation above is smaller than the mode c. This happens when the probability assigned to the open interval $\tilde{p}$ is larger than $1-[(b+1)/(b-c)]^2$. The farther to the left c is relative to the -1% truncation point, the larger should be the probability $\tilde{p}$ assigned to the open interval in order to obtain a consistent value of the lower bound a. The intuition for this result is as follows: the more pessimistic is the point forecast relative to the truncation point, the larger should be the probability assigned to the open interval to be consistent with the triangular distribution and to provide a parameter a that is smaller than c. However, it is not possible to design a triangular distribution when the forecaster assigns a significant probability mass to values larger than the truncation point and, at the same time, expects the mode (point forecast) substantially smaller than the truncation. The fact that some forecasters have point forecasts in the tails of their own density forecasts is a known result from Engelberg *et al.* (2009) and Clements (2009). One possible explanation of this phenomenon is that forecasters use an asymmetric loss function in which case it is optimal to use quantiles as point forecasts, instead of the conditional mean. An example from the ECB-SPF in the first quarter of 2009 is provided by forecaster number 2 who assigned 50% probability that real GDP growth would be between -1% and 2% and 50% below -1%. Hence, in this case we would expect the center of a unimodal distribution to be located close to -1% which is at odd with the reported point forecast of -2.5%. Using the approach I discussed above, would lead to set $b = 2$ and $c = -2.5$ and the formula above would provide a value of a equal to -2%. In these cases, the assumption that the mode of the triangular distribution is equal to the point forecast is inadequate and we follow a different approach that is discussed below.

To solve this problem, when $\tilde{p} \leq 1-[(b+1)/(b-c)]^2$ I assume that the point forecast represents the τ quantile of the triangular distribution. I set $\tau = (\mu - a)^2/[(b-a)(c-a)]$, where τ is set equal to 0.20, μ represents the point forecast, b is chosen as above, and a is obtained by solving the quadratic equation. For the example of forecaster 2 in the ECB-SPF the value of a equals to -4.75%.

100% probability on the open interval Another non-standard situation arises when the forecaster assigns 100% to the open interval. In the first quarter of 2009 there were 3 ECB-SPF forecasters that assigned 100% probability to the open interval "< -1". In this case, I assume that b is the truncation point -1%, c equals the point forecast, and $a = 2c - b$. With these choices of parameter values the pseudo distribution is assumed to be symmetric and centered at the point forecast, which becomes the mean/mode/median of the distribution. While we maintain the assumption that c equals the point forecast, the point and density forecasts do not provide enough information to identify both a and b without introducing further assumptions or relying on the properties of the triangular distribution. Hence, our choice of anchoring the right tail to the truncation point ($b = -1$) and assuming symmetry to obtain a. As mentioned earlier, there are only 3 cases of forecasters assigning 100% probability to the open interval in the ECB-SPF and none in the US-SPF so that the solution proposed here should not have any remarkable effect on the results of the paper.

Figure (1) shows the density forecasts for real GDP growth provided by 25 forecasters that participated to the ECB-SPF in the first quarter of 2009 together with the pseudo-histograms constructed with the methodology discussed above. The dot in the graph represents the point prediction that the forecaster made in the same quarter and for the same target. The effect of the adjustment is to produce, in many cases, skewed pseudo-distributions due to the fact that the point forecast is significantly shifted to the left relative to the truncation point. In Figure (2) we compare the consensus pseudo-distribution for the first and second quarter of 2009 with the histogram of the point forecasts provided by the same forecasters. Although the two objects are quite distinct, they provide a similar picture in terms of the changing views of forecasters about the state of the economy. Instead, for the US-SPF the adjustment changes only moderately the histogram provided by the forecaster and most of the quantities derived from the distributions, such as the mean and standard deviation, do not display any significant difference. In Figure (3) I show the average precision of the signal for the two Surveys over time when the density forecasts are corrected for the boundary problem. Relative to the earlier Figure for the signal precision, the biggest change occurs in the ECB-SPF, where the big drop in the first quarter of 2009 disappears and the average precision is mostly positive.

2 A measure of surprise and revisions

In quarter q of year t the forecaster observes the newly released data for quarter $q-1$ in the case of the US-SPF, and for quarter $q-2$ for the ECB-SPF. I denote by $Y_{q-j,t}$ the value of the real output and price indices that are released in quarter q, with $j = 1$ or 2 depending on the Survey that is being considered. A measure of surprise of the release content could be defined by comparing $Y_{q-j,t}$ to the distribution provided by a forecaster in quarter $q-j$. However, the quarterly release of the level of the variable is not directly comparable to the density forecasts which refers to the percentage annual change of the average value of the variable for the US-SPF (i.e., $y_t = \bar{Y}_t/\bar{Y}_{t-1} - 1$) and the fourth quarter annual percentage change of the variable for the ECB-SPF (i.e., $y_t = Y_{4,t}/Y_{4,t-1} - 1$). To transform the quarterly release of the variable to the appropriate growth rate I use the following approach which, to simplify the discussion, I describe in the specific case of a US-SPF forecaster in the third quarter of year t.

In the second quarter the forecaster observes $Y_{1,t}$ and forms expectations about the realizations for the rest of the year in order to calculate y_t. In the following quarter the forecaster observes $Y_{2,t}$ and revises the density forecast to incorporate the new information available. Ideally, we would like to measure the surprise content of $Y_{2,t}$ relative to the forecaster's expectation in quarter 1. Unfortunately we only observe the density forecast for the annual growth rate y_t rather than the quarterly growth rate. Hence, I construct an expectation for y_t, denoted by $\tilde{\mu}_{i,2,t}$, as follows:

1. In quarter 2 the forecaster knows $Y_{1,t}$ and has an expectation of growth of the variable in the remaining three quarters of $m_{2:4,t} = (1 + \mu_{2,t}) - Y_{1,t}/(4\bar{Y}_{t-1})$, with the second term indicating the growth rate that has already realized in the first quarter.

2. From the growth rate expected for the following three quarters, $m_{2:4,t}$, I extract an implied quarterly rate $m_{2,t}$ that I obtain by finding the value that minimizes:

$$\left(m_{2:4,t} - \frac{(1 + m_{2,t})^4 - (1 + m_{2,t})}{m_{2,t}} \right)^2$$

where $[(1+m_{2,t})^4 - (1+m_{2,t})]/m_{2,t} = \sum_{j=1}^{3}(1+m_{2,t})^j$.

3. I then use the implied growth rate expected in quarter 2, $m_{i,2,t}$, to calculate $\tilde{\mu}_{i,2,t}$ as follows:

$$\tilde{\mu}_{2,t} = \frac{Y_{1,t} + Y_{2,t} + Y_{2,t}(1+m_{2,t}) + Y_{2,t}(1+m_{2,t})^2}{4\bar{Y}_{t-1}} - 1$$

   This quantity represents the annual growth rate that forecaster i would have expected in quarter 2 if he/she knew the realization for that quarter, $Y_{2,t}$.

4. The surprise contained in the macroeconomic announcement in quarter 2 is thus given by $S_{3,t} = (\tilde{\mu}_{2,t} - \mu_{2,t})/\sigma_{2,t}$, where the numerator represents the surprise or news in the second quarter release relative to the expectation formed in the previous quarter, and the denominator represents the standard deviation that was expected by the forecaster in the previous quarter.

This measure can be interpreted as a standardized measure of the data release using the mean and standard deviation of the density forecast in the previous quarter. A large (absolute) value of the surprise indicates that the newly released data was considered unlikely by the forecaster based on the previous quarter density forecast.

The approach described above to measure the surprise applies also to the case of the ECB-SPF with two exceptions. The first is that real GDP for the euro-area is published with a lag of two quarters, although the HICP price index is published monthly so that the lag is one quarter. The second difference is that the object being forecast is the 4-quarter percentage change rather than the average-over-average. This simplifies the second step since there is no need to solve the quadratic equation and $m_{2,t}$ is simply obtained as $m_{q,t} = (1+\mu_{q,t})^{\frac{1}{4}} - 1$ for $q = 1$ and 2, while in quarter 3 and 4 we need to take into account that the first and second quarter, respectively, have been released. In quarter 3 I calculate the quarterly growth rate as $m_{3,t} = (\frac{1+\mu_{3,t}}{Y_{1,t}/Y_{4,t-1}})^{\frac{1}{3}} - 1$ and in quarter 4 as $m_{4,t} = \left(\frac{1+\mu_{4,t}}{(Y_{1,t}Y_{2,t})/Y_{4,t-1}^2}\right)^{\frac{1}{2}} - 1$. With the $m_{q,t}$ I am able to calculate the $\tilde{\mu}_{q,t}$ and the surprise in quarter q denoted by $S_{q,t}$ as defined above.

2.1 Binary model

The analysis in the previous Section has focused on explaining the magnitude of the weight rather than the non-Bayesian behavior of forecasters that, occasionally, become more uncertain despite having observed more information closer to the target date. I thus extend the analysis in the previous Section by considering, as dependent variable, the binary event that takes value 1 if $\rho_{i,q,t}$ is larger than 1 and 0 when $0 \leq \rho_{i,q,t} \leq 1$. I use a probit panel model and consider the same variables that I used in the previous Section. I consider only a pooled ML estimator since there are no theoretical results available on the application of the grouped estimator to limited dependent variable panel data models. Another issue is that the methods available to account for cross-sectional dependence have not been extended to this type of models. However, Hsiao *et al.* (2012) shows that it is possible to test the hypothesis of cross-sectional independence using the CD test of Pesaran (2004) discussed earlier applied to the Pearson residuals of the model. These residuals are defined as $[I(\rho_{i,q,t} > 1) - \Phi(\beta' \tilde{X}_{i,q,t})]/[\Phi(\beta' \tilde{X}_{i,q,t})(1 - \Phi(\beta' \tilde{X}_{i,q,t}))]$, where $\Phi(\cdot)$ represents the normal CDF and $\tilde{X}_{i,q,t}$ includes the quarterly dummy variables $Q3_{q,t}$ and $Q4_{q,t}$ as well as the vector of independent variables $X_{i,q,t}$ defined earlier. With these caveats in mind, I think it is still relevant to consider whether these variables have an effect in explaining the occurrence of the non-Bayesian behavior rather than being mostly driven by the Bayesian updating of the density forecasts.

The results for the output and inflation variables in the two Surveys are provided in Table (1). The results indicate that the probability of non-Bayesian behavior increases following a positive surprise (US-SPF), and a negative surprise (US-SPF PGDP and ECB-SPF GDP). In all cases, a smaller number of bins in the previous quarter (i.e., narrow prior density) increases the probability of the weight exceeding the threshold value. These results are broadly consistent with the earlier evidence, although for the ECB-SPF the effect of surprises seems to be less relevant ? to the weight regressions. In terms of cross-sectional dependence, the CD test does not reject the null hypothesis of independence for the US-SPF variables, but it does reject for the ECB-SPF forecasts. However, the magnitude of the average correlation shown in the Table is approximately 0.02 which is quite small to expect significant effects on our coefficient estimates from neglected cross-sectional dependence.

Table 1: **Probit Regression**

Variable	US-SPF		ECB-SPF	
	PGDP	GDP	HICP	GDP
$\rho_{i,q-1,t}$	**-0.29**	**-0.54**	**-0.23**	**-0.61**
p-value	0.02	0.00	0.06	0.00
$S^{+}_{i,q,t}$	**0.51**	**0.39**	0.16	0.09
p-value	0.06	0.03	0.17	0.13
$S^{-}_{i,q,t}$	**-0.45**	0.06	-0.15	**-0.13**
p-value	0.05	0.78	0.27	0.02
$Bins_{i,q-1,t}$	**-0.10**	**-0.10**	**-0.12**	**-0.14**
p-value	0.03	0.03	0.00	0.00
R-square	0.63	0.61	0.56	0.57
CD	0.14	0.43	2.93	3.79
Av. Corr.	0.001	0.003	0.02	0.023

The dependent variable is $I(\rho_{i,q,t} > 1)$ in a pooled probit panel FE regression with the same independent variables considered earlier. $R-square$ represents the goodness-of-fit statistic, and CD the cross-sectional dependence test of Pesaran (2004) applied to the Pearson residuals, while the $Av.\ Corr.$ indicates the average pairwise correlation coefficients of the residuals.

References

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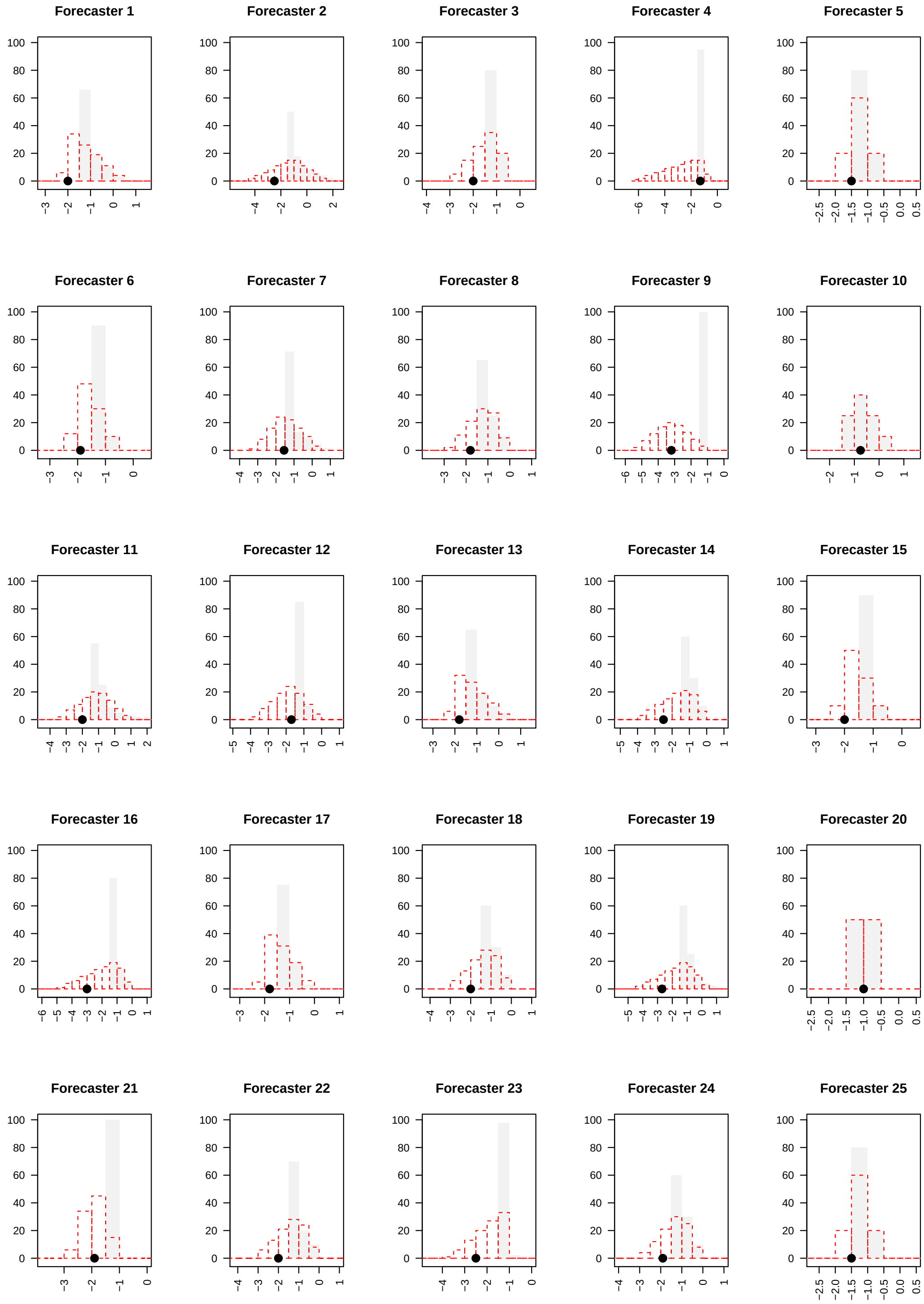
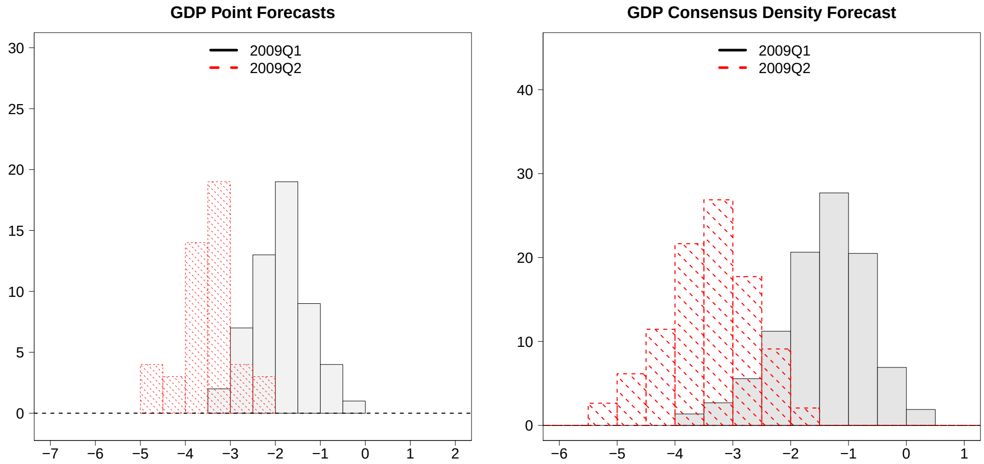
Pesaran, M. Hashem (2004). General diagnostic tests for cross section dependence in panels. *Working paper.*

Figure 1: **Pseudo-Histograms for the ECB-SPF**



Histograms for the first quarter of 2009 as provided in the Survey (gray bars) and the pseudo-histograms (dashed red lines) for the first 25 forecasters that participated to the ECB-SPF survey in that quarter. The object being forecast is real growth of the Euro-area GDP. The dot indicates the point forecast provided by the forecaster in that quarter and for the same horizon.

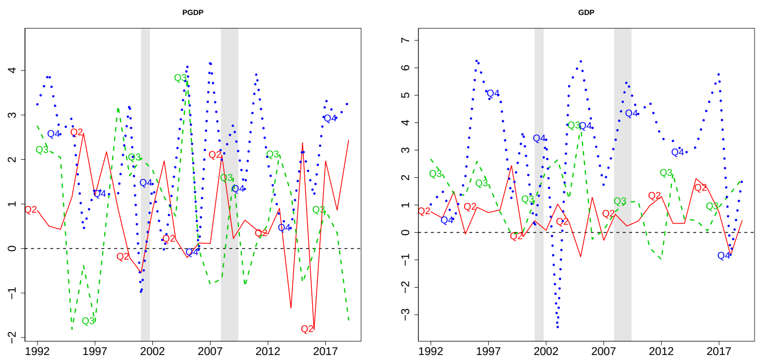
Figure 2: **ECB-SPF (2009 Q1 and Q2)**



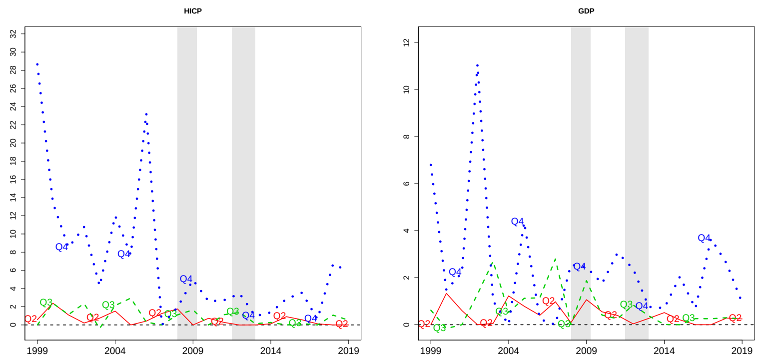
Histogram of the point forecasts (*left*) and consensus pseudo-density forecasts (*right*) for the first and second quarter of 2009 (gray bars and red dashed lines, respectively).

Figure 3: **Average Signal Precision by Quarter over Time (Pseudo histograms)**

US-SPF



ECB-SPF



Cross-sectional average of the signal precision obtained as the difference between the prior and posterior precisions in each quarter. The precision has been calculated on the pseudo-density forecasts that correct for the boundary problem. The shaded areas represent the NBER recession periods. The top two graphs refer to the US-SPF and the bottom graphs refer to the ECB-SPF.